

Three-wave mixing with three incoming waves: Signal-Idler Coherent Cancellation and Gain Enhancement in a Parametric Amplifier

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Coherent, purely-dispersive three-wave mixing systems in optics and superconducting microwave circuits can be operated as parametric amplifiers, generating from a pump wave at one frequency amplified signal and idler waves at lower frequencies. Here we demonstrate the reciprocal process using a Josephson amplifier in which coherently imposed signal and idler beams up-convert to the pump frequency. For signal and idler beams strong enough to significantly deplete the pump, we show that this reciprocal process (“coherent cancellation”) leads to large, phase-sensitive modulation and even enhancement of the amplifier gain, in good agreement with theoretical predictions.

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Parametric amplification based on three-wave mixing is a fundamental process in electromagnetic signal processing [1], both in the optical and microwave frequency domain. More recently, with the advent of quantum information science, three-wave mixing provides a basic building block for measurements at the single photon level [2, 3], where it is crucial that the nonlinear mixing process is purely dispersive. An important class of parametric amplifiers make use of three-wave mixing to amplify incoming signal fields through down-conversion of a higher frequency pump field. The amplification process involves incoming pump photons at angular frequency ω_P being split up into outgoing signal and idler photons at frequencies ω_S and ω_I respectively, where $\omega_P = \omega_S + \omega_I$. The three-wave mixing equations for the photon fields leads, under the undepleted (stiff) pump approximation, to a linear two-port scattering matrix for the signal and idler fields [4]. As recently emphasized [5], the symmetry of the three-wave mixing equations at the classical level implies that the parametric process can be operated in reverse, converting signal and idler photons, in presence of the pump, into additional pump photons. We refer to this reversed process as coherent cancellation (CC). Unlike the typical amplification process, in which only signal and pump beams are present as inputs, coherent cancellation requires three coherent input beams: along with the pump, both signal and idler must be present and balanced in amplitude, and there must be a specific phase relation of the three beams. In this case, and in contrast to a matched termination where the power is absorbed and converted into heat, all of the incident signal and idler power undergoes CC and reappears at the pump port. Hence, the pump oscillation inside the device is enhanced by signal and idler, and the gain can actually increase beyond its undepleted value.

The Hamiltonian of a three-wave mixing device [6, 7], under the rotating wave approximation (RWA), neglect-

ing external drive and signal fields, is

$$H^{\text{RWA}} = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\omega_c c^\dagger c + \hbar g_3 (a^\dagger b^\dagger c + abc^\dagger), \quad (1)$$

where a , b , and c are the annihilation operators of the signal, idler, and pump modes of center frequency ω_a , ω_b , ω_c , and bandwidth κ_a , κ_b , κ_c , respectively, and g_3 is the coupling strength. The term $a^\dagger b^\dagger c$, exploited for amplification (annihilation of a pump photon for the creation of a pair of signal and idler photons), is accompanied by its counter-part abc^\dagger , which describes the new operation that can be seen as the reciprocal of amplification (annihilation of one signal and idler photon together leading to the creation of a pump photon). The work we present here reveals this process demanded by the time-reversal symmetry of the Hamiltonian, (1). We observe this both by measuring the attenuation of the signal and idler beams when their relative phase is tuned to the coherent cancellation condition, and, more directly, by observing gain increase at the CC point by its effect on an additional probe tone. The latter effect is very difficult to observe in almost all practical amplifiers. Practical devices are designed with parameters optimized for gain, bandwidth, and stability, resulting in vastly different scattering properties of pump and signal/idler modes. Signal and idler powers and bandwidths are typically several orders of magnitude lower than those of the pump, so that subtle effects in the pump dynamics are hidden under a large background field, and thus not observable in lossy three-wave mixing systems. However, with the advent of superconducting Josephson amplifiers operating at the quantum limit [8–11], we possess sufficient control of all relevant degrees of freedom to observe reverse parametric effects. The CC effect and gain enhancement demonstrated in this paper are semi-classical in nature, but the effects could also be observed in the full quantum regime, where zero-point fluctuations of the fields would dominate.

Our device is a widely tunable Josephson Parametric Converter (JPC) [11, 12], which can be operated

(CPA) [15, 16]. Contrary to a CPA however, here the two input beams have different frequencies, and instead of absorption to an unspecified dissipative sink, here the input photons are converted into pump photons at their sum-frequency.

To observe CC in the JPC we slowly vary the phase ϕ between signal and idler input tones at a rate of 0.1 Hz, by offsetting the idler tone above the JPC idler mode center frequency by $\delta f = 0.1$ Hz. The parametric amplification process up-converts and amplifies the idler tone, which, as a result of the detuning, appears δf below the amplified signal tone. To be able to monitor the power at the signal port in time with sufficient resolution of the phase $\phi = 2\pi\delta f \cdot t$, we set the SA to a RBW that is faster than the detuning δf : 36 Hz when observing the attenuation of the signal tone and 51 Hz when monitoring the probe gain [17] (see Supplemental Material at [URL will be inserted by publisher] for choice of bandwidths and detunings). Fig. 2 shows the CC of the input signal tone, for signal and idler input powers well below (at least 10 dB) the 1 dB amplifier compression point [18]. The light blue trace is the measured, normalized signal output power P , as function of the relative phase ϕ between signal and idler. In addition, the data is fit to Eq. 3 which is then plotted as the overlaid white curve; the single fit parameter, $G_0 = 11$ dB agrees well with its independently measured value. As noted, at large gains, and $\phi = (2n + 1)\pi$, $n \in \mathbb{Z}$, the wave amplitudes interfere constructively to produce a normalized power output 6 dB (factor of 4) above the signal output power with idler tone off ($G_0 P_{\text{in}}$, pink trace). CC manifests itself at relative phase $\phi = 2\pi n$, when only a fraction $(4G_0)^{-1}$ of the incident power leaves the JPC through the signal and idler ports (normalized power $P = (4G_0^2)^{-1}$). The data in the inset in Fig. 2(a) shows P for increasing G_0 , confirming this behavior.

The coherent cancellation is a measure of the efficiency of the conversion of signal and idler photons into pump photons. Fig. 2(b) shows that there is good agreement between our data (blue dots) and the prediction of Eq. 3 (red) for all gains up to the experimental limit imposed by the system noise floor (black triangles; see arrow labeled ‘dist. to NF’ in Fig. 2(a)). All error bars are calculated from the noise floor data according to the Dicke radiometer formula [19]. The increase for the CC data at larger gains is due to the fact that we systematically decrease the signal and idler tone powers at larger gains to make sure to stay well below the saturation point of the device (we keep $G_0 \cdot \dot{n}_S$ approximately constant while ensuring that we exactly have $\dot{n}_S = \dot{n}_I$). This leads to a decrease in signal-to-noise ratio (SNR), as the JPC noise (and thus the system noise) increases with G_0 . A more serious limitation to the CC measurement is the fact that the noise floor (black trace in Fig. 2(a)) is pushed up when G_0 increases, while the input signal is adjusted to decrease. This means that our ability to observe the CC

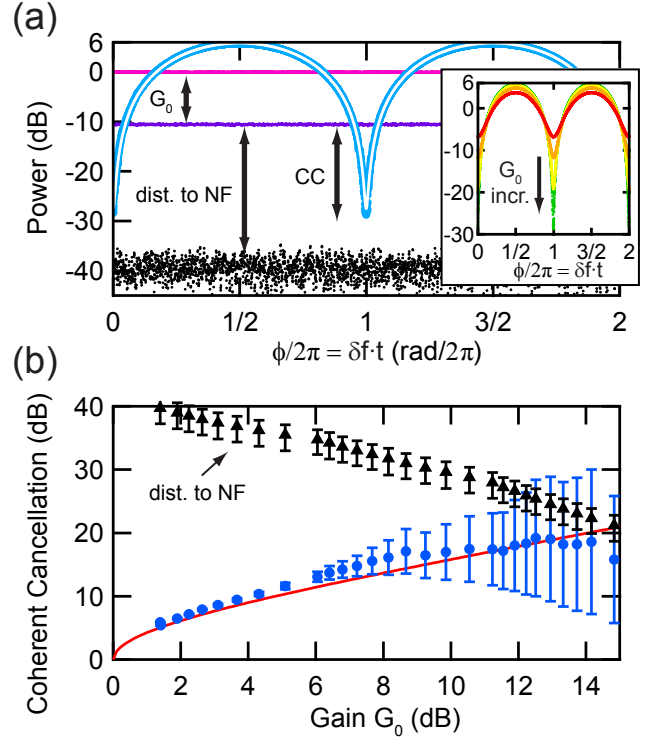


FIG. 2. (a) Normalized output power as a function of ϕ (light blue) and fit to Eq. 3 (white), for balanced signal and idler incoming photon fluxes ($\dot{n}_S = \dot{n}_I$) and $G_0 = 11$ dB. Calibration traces are as follows: idler turned off (pink); idler and pump off (purple); signal and idler off (i.e. noise floor, black). Arrows indicate how various quantities (gain (G_0), coherent cancellation (CC), distance of zero cancellation to noise floor (dist. to NF)) are measured. Inset: shown traces correspond to gains varying from 1 (red) to 11 dB (green). (b) Magnitude of CC versus gain at maximum interference point (blue). Red solid trace represents the theory, and the black triangles the resolution limitations due to the finite distance to the noise floor.

effect decreases for increasing gains (see ‘dist. to NF’ in Fig. 2(b)). This could of course be improved by reducing the RBW of the spectrum analyzer (pushing down the noise floor), but at the expense of having to perform the experiments slower by the same factor to keep the current phase resolution, which would make it more sensitive to $1/f$ noise and phase drifts.

The coherent cancellation effect as described by Eq. 3 is easily calculated in the undepleted pump approximation, but it is more general and still applies in the regime of larger signal and idler relative to the pump, when the depletion or enhancement of the pump is significant. The gain modulation effects in this regime provide a method to confirm that the attenuated signal and idler photons at the CC condition are not lost to some other dissipative process, but are being coherently converted to pump photons. Specifically, as the relative phase, ϕ of the signal

and idler is varied, and the JPC is alternatively amplifying and attenuating these inputs, the pump is either depleted or strengthened, and this can be observed as a phase dependent modulation of the effective gain experienced by the weak probe tone, which we now introduce.

Experimentally, we keep the signal and idler photon fluxes equal ($\dot{n}_S = \dot{n}_I$) but in contrast to the experiment of Fig. 2, with signal and idler amplitudes large enough to significantly saturate the device. Eq. 3 still holds, with the undepleted gain, G_0 replaced by a non-linear gain, $G = G(\phi, x)$, $x = \dot{n}_S/\dot{n}_P$, which must be calculated self-consistently. Fig. 3(a) shows the expected

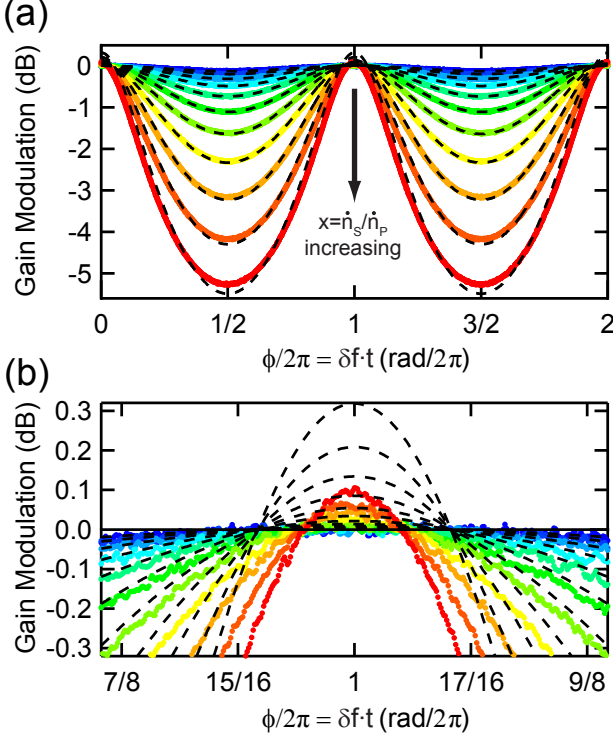


FIG. 3. (a) Phase-dependent depletion of signal port gain ($G_0 = 11$ dB). Amplified probe power is measured in a 51 Hz RBW (signal tone is offset by 100 kHz). Traces correspond to signal and idler photon fluxes increasing by 2 dB (from blue to red). Dashed lines are least square fits of gain modulation theory. (b) Enlargement of the region around $\phi = 2\pi$. Gain enhancement above $G_0 = 11$ dB (0 dB line) is clearly visible for experimental data.

large modulation of the gain with ϕ , and, most dramatically, around $\phi = 2\pi$ (see Fig. 3(b)), we observe an *increase* in the JPC gain; moreover, the gain enhancement increases as we increase x in 2 dB steps (colors from blue to red). The data set shown corresponds to an undepleted gain of 11 dB (normalized to 0 dB line), and an applied pump power that is not changed across traces. Note that the gain enhancement $\sim x$, whereas the gain depletion $\sim 4G_0x$, so one expects the enhancement to be smaller than the depletion.

We can calculate $G = G(\phi, x)$ by solving the equations of motion derived from the three-wave mixing Hamiltonian (Eq. 1) iteratively in the correction to the undepleted pump approximation (see Supplemental Material at [URL will be inserted by publisher] for details on derivation). This leads to the self-consistent equations for the pump parameter $\rho \in [0, 1]$:

$$\rho = \rho_0 \left[\left[1 - i \frac{\rho_0}{4} x \frac{1}{\sqrt{G}} \left\{ (1 + \sqrt{G})^2 e^{i(\phi + \varphi_p)} - 2i(1 + \sqrt{G})\sqrt{G-1} - e^{-i(\phi + \varphi_p)}(G-1) \right\} \right] \right], \quad (4)$$

$$\sqrt{G} = \frac{1 + \rho^2}{1 - \rho^2}, \quad (5)$$

where $\rho_0 = \sqrt{(G_0 - 1)/(G_0 + 1)} \in [0, 1]$ is the undepleted pump parameter, and G_0 is the undepleted gain (no signal/idler inputs). The dashed lines in Fig. 3 correspond to theory with a single fit parameter x . As expected, the fits reproduce the 2 dB steps of applied signal and idler powers (not shown). We find excellent agreement between our theory and our experiment for phases ϕ away from $2\pi n$, where signal and idler fields constructively interfere and lead to a depletion of the pump photon flux, manifested as a decrease of the JPC gain.

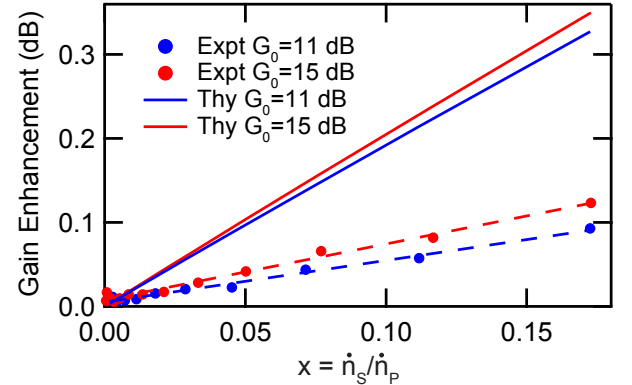


FIG. 4. Experimental gain enhancement data (circles) and theory (lines) for $G_0 = 11$ dB (blue) and $G_0 = 15$ dB (red). The gain enhancement is evaluated at a relative phase ϕ between signal and idler tones of 2π and plotted against photon flux ratio x .

Figure 4 shows a direct comparison of the experimental gain enhancement to the theoretical prediction at $\phi = 2\pi n$ as a function of $x = \dot{n}_S/\dot{n}_P$ for gains of 11 dB and 15 dB. Although the experimental slope is about 1/3 of the expected theoretical slope, we unambiguously observe significant gain enhancement in our JPC, the key signature of the coherent conversion of signal and idler photons into pump photons. There are several reasons for the discrepancy between experimental and theoretical gain enhancement: the experiment requires significant averaging with the given bandwidths and powers

(ultimately limited by the JPC dynamical bandwidth and dynamic range), while each modulation is rather slow (0.1 Hz). It is thus sensitive to $1/f$ noise and microwave generator phase drifts. Further, any mismatch between \dot{n}_S and \dot{n}_I decreases the gain enhancement. Finally, there may be a contribution due to spurious dissipation in the CC process.

The presented gain enhancement results can be understood as a benchmark for the level of control we have over all degrees of freedom in our three-wave mixing device. Any hidden (uncontrolled) degree of freedom other than the signal, idler, and pump modes will inevitably be perceived as dissipation and thus lead to a reduction in the gain enhancement effect. A potential application of the reverse operation of a JPC is its use in phase-locking two coherent tones of different frequencies in a feedback loop, which could be crucial for quantum information processing with artificial atoms.

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